$$< n >= \{1,1,2,2,2,2,3,3,3,3,3,3,4,4,4,4,4,4,4,4,5,5,5,\ 5,5,5,5,5,5,5,6,...\}$$

$$\sum_{n=1}^{\infty} \frac{2^{<n>} - 2^{-<n>}}{2^n} = \sum_{n=1}^{\infty} \left(\frac{2^{<n>}}{2^n} - \frac{2^{-<n>}}{2^n} \right) = \sum_{n=1}^{\infty} \frac{1}{2^{n-<n>}} - \sum_{n=1}^{\infty} \frac{1}{2^{n+<n>}}$$

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{2^{n-<n>}} &= \frac{1}{2^{1-1}} + \frac{1}{2^{2-1}} + \frac{1}{2^{3-2}} + \frac{1}{2^{4-2}} + \frac{1}{2^{5-2}} + \frac{1}{2^{6-2}} + \frac{1}{2^{7-3}} + \frac{1}{2^{8-3}} + \frac{1}{2^{9-3}} + \frac{1}{2^{10-3}} + \frac{1}{2^{11-3}} + \dots \\
&= \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} + \frac{1}{2^9} + \frac{1}{2^{10}} + \frac{1}{2^{11}} + \frac{1}{2^{12}} + \dots \\
&= \frac{1}{1} + \frac{1}{2} \\
&+ \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} \\
&+ \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} \\
&+ \frac{1}{2^9} + \frac{1}{2^{10}} + \frac{1}{2^{11}} + \frac{1}{2^{12}} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} \\
&+ \frac{1}{2^{16}} + \frac{1}{2^{17}} + \dots
\end{aligned}$$

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{2^{n+<n>}} &= \frac{1}{2^{1+1}} + \frac{1}{2^{2+1}} + \frac{1}{2^{3+2}} + \frac{1}{2^{4+2}} + \frac{1}{2^{5+2}} + \frac{1}{2^{6+2}} + \frac{1}{2^{7+3}} + \frac{1}{2^{8+3}} + \frac{1}{2^{9+3}} + \frac{1}{2^{10+3}} + \frac{1}{2^{11+3}} + \dots \\
&= \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^{10}} + \frac{1}{2^{11}} + \frac{1}{2^{12}} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{17}} + \dots \\
&= \frac{1}{2^2} + \frac{1}{2^3} \\
&+ \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} \\
&+ \frac{1}{2^{10}} + \frac{1}{2^{11}} + \frac{1}{2^{12}} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} \\
&+ \frac{1}{2^{17}} + \dots
\end{aligned}$$

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{2^{n-<n>}} - \sum_{n=1}^{\infty} \frac{1}{2^{n+<n>}} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^9} + \frac{1}{2^9} + \frac{1}{2^{16}} + \frac{1}{2^{16}} + \frac{1}{2^{25}} + \dots \\
&= 1 + 2\left(\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^9} + \frac{1}{2^{16}} + \frac{1}{2^{25}} + \dots \right) \\
&= 1 + 2 \sum_{n=1}^{\infty} 2^{-n^2}
\end{aligned}$$

$$\sum_{n=1}^{\infty} 2^{-n^2}$$

θ fonsiyonudur ve $q = 2$ icin yaklasik degeri

$$\sum_{n=1}^{\infty} 2^{-n^2} = 0.564468413605938579...$$

$$\sum_{n=1}^{\infty} \frac{2^{<n>} - 2^{-<n>}}{2^n} = 1 + 2(0.564468413605938579) = 2.12893682721187...$$

Eger soru soyle olsaydi tam deger cikiyor. Acaba basim hatasi mi var?

$$\sum_{n=1}^{\infty} \frac{2^{<n>} + 2^{-<n>}}{2^n} = \sum_{n=1}^{\infty} \left(\frac{2^{<n>}}{2^n} + \frac{2^{-<n>}}{2^n} \right) = \sum_{n=1}^{\infty} \frac{1}{2^{n-<n>}} + \sum_{n=1}^{\infty} \frac{1}{2^{n+<n>}}$$

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{2^{n-<n>}} + \sum_{n=1}^{\infty} \frac{1}{2^{n+<n>}} &= \frac{1}{1} + 2\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} + \frac{1}{2^{10}} + \frac{1}{2^{11}} + \dots \right) \\
&= 1 + 2\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1 + 2\left(\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - 1 \right) \\
&= 1 + 2\left(\frac{1}{1 - \frac{1}{2}} - 1 \right) \\
&= 3
\end{aligned}$$